Adaptive Cell Tower Location Using Geostatistics

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In this article, we address the problem of allocating an additional cell tower (or a set of towers) to an existing cellular network, maximizing the call completion probability. Our approach is derived from the adaptive spatial sampling problem using kriging, capitalizing on spatial correlation between cell phone signal strength data points and accounting for terrain morphology. Cell phone demand is reflected by population counts in the form of weights. The objective function, which is the weighted call completion probability, is highly nonlinear and complex (nondifferentiable and discontinuous). Sequential and simultaneous discrete optimization techniques are presented, and heuristics such as simulated annealing and Nelder–Mead are suggested to solve our problem. The adaptive spatial sampling problem is defined and related to the additional facility location problem. The approach is illustrated using data on cell phone call completion probability in a rural region of Erie County in western New York, and accounts for terrain variation using a line-of-sight approach. Finally, the computational results of sequential and simultaneous approaches are compared. Our model is also applicable to other facility location problems that aim to minimize the uncertainty associated with a customer visiting a new facility that has been added to an existing set of facilities.

Introduction and literature review

This article is motivated by the challenge of optimally locating new base stations for wireless service, using data on existing cell phone signal strength measurements,
geographic variation of demand, and topography. A wireless communication system is characterized by (1) a number of base stations, (2) an area over which base stations receive and transmit a radio signal, which can be determined by a known radius and topography, (3) a signal strength, and (4) a spatial distribution of demand for wireless service (Lee 1985; Krzanowski and Raper 2001). Accurately locating base stations is crucial to the quality of the wireless communication system and to satisfying the demand of clients, which varies in space. Several external factors affect the quality and strength of a signal, including slope, altitude, foliage, obstruction and distance to a cell tower (Delmelle et al. 2005). Finding the optimal locations for a fixed number of base stations under constraints of capacity (channel and radius), required signal quality, and adequate demand coverage, is a challenge, especially in rural areas (Akella et al. 2005).

Facility location design
Facility location problems such as base station placement commonly occur because a number of facilities must be located in a region to satisfy customer demand. A specified objective function is used to select the optimal set of base station locations from the set of all potential sites. Some of the objectives considered in the literature are to minimize the number of facilities, to minimize distance to a facility, and to maximize coverage of customer demand. An assumption underlying all coverage models is that customers beyond a specified service range are not adequately served by the service facilities in question.

However, what determines whether a customer chooses to visit a facility (e.g., grocery store, bank) is not just distance, but several other factors, including but not limited to, accessibility, convenience, competitor locations, travel, waiting time, and traffic conditions en route (Brandeau and Chiu 1994). Distance is by far the most studied factor, where a common objective is to minimize the total demand-weighted travel distance between demand nodes and retail facilities (Hakimi 1964). A modified version of this $p$-median problem in the presence of competing firms locates facilities to maximize the number of new customers captured (ReVelle 1986). The $p$-center or minimax problem is another distance-based model minimizing the maximum distance between any demand point and its nearest facility (Drezner 1995a, b). Although indirectly a function of distance, the set covering problem (SCP) finds a minimum number of facilities in a way that every demand node is covered by at least one facility (Daskin 1995).

Often the influence of competitors is modeled using a game theoretic framework (Ghosh and Craig 1983; De Palma et al. 1989). Huff (1963) introduces a probabilistic model predicting consumer spatial behavior and explicitly evaluates the probability that a customer selects particular facilities from among a set of existing ones. This probability depends on the utility of a facility (e.g., square footage of a store) relative to the utilities of and distances to others and on the distance between a customer and the facility. In unconstrained gravity models, the attraction of a customer to a facility is proportional to the size of the facility and
inversely proportional to the square (or some other power) of the distance to the facility. Fotheringham (1983) points out that this approach does not include the accessibility of the facility relative to all other possible destinations: as the accessibility of a facility to all other facilities increases, the attraction of a customer to that facility decreases.

**Augmenting an existing set of facilities**

Facility location models are design problems in the sense that initially no facilities exist. Augmenting a set of facilities has been addressed by Ostresh (1978), Berman and Simchi-Levi (1990), Drezner (1995b), and Wang et al. (2003). Ostresh introduces a stepwise location-allocation problem, which finds the location of an additional facility by minimizing the aggregate weighted distance from all demand points. Suzuki, Asami, and Okabe (1991) use a myopic approach to locate sequentially one additional facility at a time under four different policies. The first policy, which is termed the socially optimal allocation of facilities, is very relevant to this article. A new facility is constructed in each time period. Because all facilities are similar, all customers choose the closest one. The model sequentially locates new facilities by minimizing the sum of weighted distance, where the weights reflect population density.

**Cell Tower Placement**

The adaptive base station positioning algorithm (ABPA) uses an early version of the demand node concept, which captures the spatial distribution of cell phone service demand by using a set of discrete points (Fritsch, Tutschku, and Leibnitz 1995). The nodes constitute a static population model describing the demand density surface. The model reduces to a maximal covering location problem (MCLP) (Szabó, Weicker, and Widmayer 2002). A major drawback of ABPA is its lack of speed, and Wright (1998) suggests a direct search method because it requires only the value of the function to be optimized. Bose (2001) uses dynamic programming to determine an optimal placement for new base stations in an urban setting, given current cell coverage. Mathar and Niessen (2000) present a variety of mixed-integer base station placement problems in which the locations and configurations of base stations are chosen so that most demand nodes (in the form of traffic points) are served and that, at the same time, interference and multi coverage are kept small. Krzanowski and Raper (2001) design an entire wireless communication system using a spatial evolutionary algorithm (SEA), based on genetic algorithms, following the structure of the MCLP defined by Church (1984). However, this model does not account for topography, and demand is uniformly distributed within the region under study, generating a very systematic pattern of transmitters. Because evolutionary search algorithms can break out of local optima by introducing randomness, the results outperform both random and greedy (hill-climbing) algorithms. Chamberland (2004) suggests a model for expanding an existing wireless network while minimizing the expansion cost of the existing network subsystem, keeping a specific level of network performance. Wang et al. (2003) consider a more general model that allows for both addition and deletion of facilities. Finally, Akella et al. (2005)
present the network design emergency coverage model, a mixed-integer program-
mixed-integer programming model that integrates population demand and frequency channel assignment, and that covers requirements of high-crash locations.

The contribution of this article is threefold. First, the placement of additional base stations follows an adaptive approach using kriging, given that it is explicitly governed by cell phone signal strength data measured in the region of interest. Second, the method suggested here to locate additional towers using kriging and uncertainty can be generalized to other facility location problems when the probabilities of visiting these facilities are known. Third, landscape morphology is accounted for by using a line-of-sight approach. This last contribution is important because most cell tower location models ignore terrain variation.

Kriging and the adaptive spatial sampling problem

The method developed in this article directly uses the probability of cell phone call completion data at certain discrete locations. A commonly accepted measure of the strength of the bond between a cell phone and a cell tower (and consequently a call completion probability) is the relative signal strength indicator (RSSI). The probabilities of call completion are calculated for given RSSI levels (see Akella et al. 2003; Delmelle et al. 2005 for a general discussion on RSSI and factors affecting signal strength).

Kriging is a geostatistical interpolation technique accounting for the presence of spatial autocorrelation among sample observations (Cressie 1991; Bailey and Gatrell 1995). It has been applied in the field of adaptive sampling to determine where additional measurements should be made to reduce the uncertainty associated with the spatially distributed phenomenon being estimated (Thompson and Seber 1996). The procedure for selecting additional samples is a function of the outcome of the variable of interest, as observed during an initial sampling phase of a survey (Makarovic 1973; Ayeni 1982). The addition of new samples improves the confidence for sample-based estimates (Delmelle and Goovaerts 2009). Adaptive sampling techniques also have been used in the context of soil contamination (Cox 1999). Rogerson et al. (2004) develop a method to locate additional sampling locations, which are determined in a sequential fashion that minimizes the weighted prediction error. The weights increase in regions of high accident densities and varying cell phone signal strength. Another application of this approach is that of a cellular provider faced with the problem of locating an additional cell tower to improve coverage in a region.

In this article, we use sampled cell phone signal strength data and geostatistics to locate sequentially a set of additional base stations to maximize the completion probability of a randomly placed call. We use simple kriging as a weighted linear interpolation technique that estimates the value of a variable at unknown locations based on observed values at prespecified locations.
The following equation predicts a variable $z$ at location $i$ as a linear combination of the values observed at surrounding points $j$ (Goovaerts 1997):

$$
\hat{z}(i) = \mathbf{\lambda}_j^T(i) \cdot \mathbf{z} = \mathbf{c}^T(i) \cdot \mathbf{C}^{-1} \cdot \mathbf{z}
$$

where $\mathbf{z}$ denotes the vector of $z(j)$ values, and $\mathbf{\lambda}_j^T(i)$ denotes the vector of weights associated with the observations $j$ in the neighborhood of $i$. Here, $z(j) = p(j)$, which represents the probability of call completion at sample site $j$, while $\hat{z}(i)$ is the predicted probability of call completion at location $i$. The weights are chosen so that the expected mean squared prediction error across all points (i.e., the kriging variance) is minimized. Adaptive sampling optimally locates supplementary samples in a way that minimizes the uncertainty in the estimates. In simple kriging, the variance at a point $i$ is defined as

$$
\sigma^2_k(i) = \sigma^2 - \mathbf{c}^T(i) \cdot \mathbf{C}^{-1} \cdot \mathbf{c}(i)
$$

where $\sigma^2$ is the sill (i.e., the semivariance value corresponding to the range). The use of the simple kriging model does not affect the behavior of the kriging variance, which increases away from existing sample points and equals zero at existing data points. Therefore, the optimal location of a new sample point that minimizes the kriging variance at a single point $i$ is $i$ itself. The total kriging variance (TKV) is obtained by integrating equation (2) over a study area $D$. This is made easier computationally by superimposing a regular grid on $D$ and summing the kriging variance calculated for the set of grid points $G$, such that

$$
\text{TKV} = \int_D \sigma^2_k \, dD \approx \sum_{g \in G} \sigma^2_k(g) = \sum_{g \in G} [\sigma^2 - \mathbf{c}^T(g) \cdot \mathbf{C}^{-1} \cdot \mathbf{c}(g)]
$$

where $g$ denotes a grid point. We now consider the determination of the optimal location of one additional sample point $s_{\text{add}}$ such that the change in TKV over $G$ is a maximum across the new sample set $M' = M \cup \{s_{\text{add}}\}$, where $M$ is the set of initial sample observations, with cardinality $m$. This optimization problem was introduced by Cressie (1991) and discussed in Van Groenigen and Stein (1998):

$$
\begin{align*}
\text{s}_{\text{opt}} & = \max_{s_{\text{add}}} \Delta \sigma^2_k(s_{\text{add}}) = \max_{s_{\text{add}}} [\text{TKV}_{\text{old}} - \text{TKV}_{\text{new}}] \\
& = \max_{s_{\text{add}}} \left[ \sum_{g \in G} (\sigma^2 - \mathbf{c}^T(g) \cdot \mathbf{C}^{-1} \cdot \mathbf{c}(g)) - \sum_{g \in G} (\sigma^2 - \mathbf{c}^T_{s_{\text{add}}}(g) \cdot \mathbf{C}^{-1}_{s_{\text{add}}} \cdot \mathbf{c}_{s_{\text{add}}}(g)) \right]
\end{align*}
$$

where $\text{TKV}_{\text{old}}$ and $\text{TKV}_{\text{new}}$ are the TKVs calculated with the set of $m$ initial sample points and with the augmented set of $m+1$ points, respectively. The subscript $s_{\text{add}}$ denotes the addition of a new sample point. The location of the additional point is chosen from a set of candidate locations $P$, which can be discrete (in this case $P$ is a lattice) or infinite. The size of the set of candidate locations $P$ greatly influences the computational complexity and hence the need to use heuristic methods.
An analogy to the cell tower location problem

In facility location problems, an optimal set of locations among potential sites is selected according to an objective function. The same objective holds for locating additional facilities. In this section, we develop an analogy between the adaptive spatial sampling problem and the additional cell tower location problem. When applied to cell tower locations, the optimization criterion suggested in equation (4) (i.e., minimization of the kriging variance) seeks to produce an optimal cell tower location far away from existing sample towers, which is similar in nature to a dispersion criterion (Kuby 1987). However, if no information is available about existing towers, known cell phone signal strength data points furnish a possible surrogate measure. Equation (4) can be improved so that a new tower is located far away from sample measurements characterized by strong call completion probabilities. A different objective function is presented, based on the call completion probability for a randomly placed customer; this measure is derived from existing sampled cell phone signal strength data. A geostatistical model is applied to predict the probability at unknown locations based on sample observations. With the addition of a new cell tower \( \text{ct} \) equation (1) for a grid point \( g \) becomes

\[
Z_{\text{ct}}^*(g) = \lambda_{\text{ct}}^T(g) \cdot Z_{\text{ct}} = C_{\text{ct}}^T(g) \cdot C_{\text{ct}}^{-1} \cdot Z_{\text{ct}}
\] (5)

The subscript \( \text{ct} \) denotes the location of a new cell tower. A principal assumption is that the call completion probability at a cell tower equals one. The cell tower location problem considered is that of allocating one (or more) additional base station(s) to maximize the weighted completion probability of a phone call placed by any customer from a region specified in the set \( G \):

\[
\text{ct}_{\text{opt}} = \text{MAX}_{\text{ct}} \sum_{g \in G} w(g) Z_{\text{ct}}(g)
\]

\[
= \text{MAX}_{\text{ct}} \sum_{g \in G} w(g) \lambda_{\text{ct}}^T(g) \cdot Z_{\text{ct}}
\]

\[
= \text{MAX}_{\text{ct}} \sum_{g \in G} w(g) C_{\text{ct}}^T(g) \cdot C_{\text{ct}}^{-1} \cdot Z_{\text{ct}}
\]

(6)

Here \( w(g) \) represents the weight of grid point \( g \), which can be computed in a region, for instance, by taking the ratio of the population at \( g \) to the total population. Upon addition of a new cell tower, the cell phone signal at location \( i \) is expected to improve if location \( i \) is in the vicinity of the newly added cell tower. This change must be reflected dynamically. Different scenarios are possible, depending on cell tower characteristics (e.g., power, radius, tower height). Generally, one can assume that beyond a critical distance \( d_{\text{crit}} \), the influence of a new tower on signal strength is negligible.

**Search algorithms**

Performance of the following three search algorithms is investigated next: total enumeration, simulated annealing, and the simplex method (Nelder–Mead).
Total enumeration consists of evaluating which candidate points from $P$ optimize equation (6). The total enumeration approach can be prohibitive when $P$ is large. The solution by total enumeration is optimal for the sequential addition problem in the case where $P$ is a lattice (or finite).

Simulated annealings’ major advantage over other heuristics is its ability to avoid becoming trapped at local minima. Simulated annealing always accepts solutions increasing the objective function; solutions that decrease the objective function are accepted with a probability $p = e^{-\frac{\Delta f}{T}}$, which decreases as the algorithm progresses (Kirkpatrick, Gelatt, and Vecchi 1983), where $\Delta f$ is the non-improving change in the objective function $f$ and $T$ is a temperature control parameter. The temperature $T$ controls the probability of accepting nonimproving moves. Starting simulated annealing with a high temperature allows the algorithm to escape local optima. As the procedure progresses, the temperature is cooled down. The neighborhood is selected by picking a random direction in the interval $(0, 2\pi)$. The neighbor of the current solution is the new point obtained by moving a distance equal to the step size in the chosen direction. The step size is a search window around the current point, and is reduced by a factor of 1.25 after a fixed number of iterations. The new solution is accepted if it is better; if it is poorer then it is accepted with the preceding probability. Two simulated annealing versions are presented here: one searches for the location of a new tower from the finite set $P$, and one searches across a study area from the infinite set $P$.

The simplex method is a direct search technique used to optimize a nonlinear function over a finite region. The simplex method of unconstrained minimization was devised by Spendley, Hext, and Himsworth (1962) and later improved by Nelder and Mead (1965). A detailed description can be found in Avriel (1976). The simplex algorithm considers a set of points forming a simplex. From among a set of vertices, the vertex associated with the poorest function value is replaced by a new point. The replacement of this point involves one of three steps: reflection, contraction, or expansion.

**Case study**

In this case study, we try to determine the optimal location of a new cell tower that maximizes the weighted call completion probability (WCCP) of demand points. The location of the new tower should be in a region with a low call completion probability and be influenced by geographic variation in population demand. The study area, located in southern Erie County (Fig. 1a) in western New York State, is a $15 \text{ km} \times 15 \text{ km}$ area characterized by relatively large altitude variation (see Fig. 1d). The demand points are the centroids of actual census blocks (Fig. 1b) for this study area. Each of the 166 demand points is characterized by a weight equal to the population of the census block for the year 2000; these populations range from 1 to 300. To normalize it, each population value is divided by total population. Therefore, the objective function value will be $<1$. The 380 cell phone signal strength
(RSSI) data points in this region were measured along roads (Fig. 1c). Although cell phone signal strength is a variable that is continuous in space, it is less time consuming and more convenient to collect measurements on a road network (Rogerson et al. 2004). Even though samples were collected on a network, the map of cell phone signal strength is derived by using a continuous interpolation technique (kriging). This approach necessarily limits coverage, and consequently the uncertainty about the cell phone signal values increases in areas away from roads. Signal strength values serve as input data; these were measured using a modified automated crash notification device, and mapped to call completion probability values (see Akella et al. 2003 for a summary of data collection methods).

Naturally, call completion probabilities for sample points increase in the neighborhood of the added cell tower. Cell phone signal strengths were strong in the vicinity of a given cell tower, but more variable beyond a distance of 3000 m, and questionable beyond 5000 m (Delmelle et al. 2005). Therefore a conservative characterization of this landscape is given by the following equation:

Figure 1. The study area $D$ is located in the southern part of Erie County, western New York (a). Map (b) exhibits the population distribution at the block level (166 blocks). White dots represent the centroids of the census blocks. Map (c) is the interpolated call completion probability from the 380 relative signal strength indicator (RSSI) data points. Map (d) illustrates the altitude variation across the region.
where \( \text{los}(i, ct) \) is the line-of-sight between a sample point \( i \) and the proposed cell tower site. In equation (7), the term \( Z_{i,ct}^{\text{new}} \) denotes the call completion probability at location \( i \) following the addition of a cell tower \( ct \), while \( Z_{i}^{\text{old}} \) denotes the call completion probability before the addition of a new tower. The addition of a cell tower in an area characterized by call completion probabilities close to 1 will have little effect on the covariogram structure and \( Z_{i,ct}^{\text{new}} \). In cases where initial probabilities are much lower, it would be essential to recompute the covariogram to reflect the change in spatial structure in call completion probabilities following the addition of a new tower.

The line-of-sight analysis computes the mutual visibility between two locations on a displayed digital elevation map. The algorithm accounts for obstructions along the path separating any point in a study area from each cell tower (see Clarke 1995). A binary value (1 for visible and 0 otherwise) results from a line-of-sight analysis between two locations in a study region. However, the line-of-sight has some drawbacks because the signal is attenuated and reflected, depending on the presence of buildings and vegetation between a tower and a given target point. Assumptions used here include a tower height of 30 m and a target altitude of 1.5 m. This investigation also assessed solution impacts of a higher tower. The \( \text{los2} \) function in the MATLAB Mapping Toolbox was used to perform line-of-sight analysis.

Other functions that govern the relationship between signal strength and distance can be used, such as an exponential distance decay function. Once the signal strength data points were collected for the study area, the first step was to estimate the covariogram \( \tilde{C}(h) \). The range, 850 m, was used throughout our computational runs. The 15 km rectangle that bounds the study area contains the optimal location. The finite set of candidate locations \( P \) is created by discretizing the study region into 151 rows and columns (100 m spacing), generating a total of 22,801 potential locations. The possibility of locating a new cell tower at each of these potential points is evaluated by measuring its WCCP value. The optimal point (the point at which the probability is maximum) may be observed by constructing a graph of call completion probability versus a cell tower location.

Computation, results, and figures were produced using MATLAB 7.0 run on a Pentium M, with a 2.13 GHz processor and 1 GB RAM. The \( \text{fminsearch} \) function available in the MATLAB optimization library was used for the Nelder–Mead method.

**Sequential versus simultaneous addition**

A set of new cell towers may be added for computational purposes in two different ways. Either \( n \) towers are selected at one time and added to the initial set, or one
point at a time is selected and added to the initial set. The former is defined as simultaneous addition, and the latter is known as sequential addition and is suboptimal. In sequential addition, once the first tower has been found and added, \( n - 1 \) additional locations are to be chosen in a similar fashion. If the set of potential towers \( P \) has cardinality \( p \), then there are \( [p+(p-1)+\ldots+(p-n+1)] \) solutions to the problem. Sequential addition is particularly desirable when a specific level of improvement in the WCCP has to be obtained. Additional towers are located sequentially until the level of improvement has been obtained. The total enumeration heuristic is optimal in the sequential addition, but may be suboptimal in the simultaneous addition. The major concern in simultaneous addition lies in the selection of these towers. Total enumeration is not recommended in this situation because of a combinatorial explosion, \( \binom{p}{n} \), of possible solutions. Simulated annealing (infinite and discrete) and Nelder–Mead were used to solve this problem in a simultaneous fashion.

If only one tower is added, simultaneous addition is the same as sequential addition. Because of its computational efficiency, Nelder–Mead was tested with 10 different pairs of starting points (10 \( \times \) 1 different starting points in the one-tower case, 10 \( \times \) 2 different starting points in the two-towers case, and 10 \( \times \) 3 different starting points in the three-towers case). Total enumeration provides a benchmark to assess the quality of solutions obtained using other heuristic techniques.

**One additional cell tower**

We compare total enumeration with simulated annealing and the Nelder–Mead algorithms. The location of an additional tower is chosen from the set of candidate locations \( P \). The discrete set of candidate locations is relatively large in practice if the study region is discretized very finely. A finer set may result in a better solution, but at the expense of a longer running time. While total enumeration searches for the optimal cell tower location within the finite set \( P \), both Nelder–Mead and simulated annealing search over the entire study region for a potential location (infinite set). Adjustments to the simulated annealing procedure allow the search for the location of a new tower to be over the discrete, finite set \( P \).

The one-additional cell-tower case adds one cell tower within a region of interest. Because \( n = 1 \), simultaneous addition is the same as sequential addition. The surface value at a specific location represents the WCCP, given that a tower is added at that location. Figure 2 illustrates the WCCP surface resulting from the addition of one, two, and three towers, respectively. Figure 2a shows that the addition of one tower has little overall impact on the WCCP values (low values almost everywhere in the region, with the exception of a few peaks). However, in the eastern part of the study area, the addition of a tower has a significant impact. The optimal location \((x = 11,900, y = 8,000)\) is denoted by a white dot where the surface peaks at 0.87052.
Figure 2. Weighted call completion probability (WCCP) surface following the addition of the first, second, and third tower in (a), (b), and (c), respectively. In each figure, the white dot denotes the optimal location of the additional tower using a sequential enumeration with 100 m spacing. Each time a tower is added, it will augment the average WCCP surface.
Two additional cell towers
The new tower found from the one additional tower problem has been added to our set of sample points. Call completion probabilities around that first tower have been updated according to equation (7). The location of that tower has a call completion probability of 1. The search for the second tower is carried out in the same manner as the first cell tower. The surface obtained is shown in Fig. 2b, with the optimal location marked by a white dot. The WCCP values are higher than those in Fig. 2c, because we already added the first cell tower, and call completion probabilities in the vicinity of the first tower were updated. The WCCP surface drops significantly in the area where the first tower was added: adding a second tower in the vicinity of the first would make little sense. The graph (Fig. 2c) shows that multiple peaks exist with approximately similar values. The optimal location now moves relatively far from the location of the first tower.

Three additional cell towers
The second tower is added to the initial set of sample points with a call completion probability value of 1, with call completion probabilities for surrounding points again updated following equation (7). A similar approach is used to locate the third cell tower (see Fig. 2c). This time the location of the new tower moves far away from the other two added towers. Interestingly, a block emerges in the southeastern part of the study region. The same block is already perceptible in Fig. 2b, but its magnitude is now much greater. The appearance of this circular block is a result of low call completion probabilities in that region (see Fig. 1d). The algorithm did not select that area for a new tower mostly because it exhibits low population demand.

Comparison of the heuristics: Sequential addition
In sequential addition, where the set of candidate solution locations \( P \) is finite, both total enumeration and simulated annealing return the optimal solution (see Table 1). However, simulated annealing is about six times faster than total enumeration. This significant difference in computation time endorses using simulated annealing for solving realistic additional facility location problems using geostatistics. As can be expected, simulated annealing yields slightly better results when the set of candidate solution locations \( P \) is infinite rather than finite. Although Nelder–Mead takes the least amount of time, it fails to return the optimal solution for the addition of either one or two towers, probably because it uses a localized search. However, the addition of the third tower is optimal, and the overall improvement substantial. The improvement in coverage \( D_{COV} \) increases with the addition of each facility, but at a slower rate each time a tower is added.

The effect of a coarser discretization of \( P \) upon the quality of solutions using a total enumeration algorithm was analyzed. The overall quality decreases with a coarser grid, with the benefit of a much shorter running time. The decrease in running time is exponential. In the one-tower-addition case, the solution found using a 150 m interval is better than that found using a 100 m interval. However, the solution is worse in the two- and three-towers scenarios. The 50 m spacing interval
produces a set of 90,000 possible locations, and the solutions are slightly better, but approximately 10 days of computing time are necessary to find the optimal locations of the three towers. Solutions found for the 200 m spacing are slightly lower than for the 100 m spacing, and those for the 500 m spacing are significantly lower than for the 100 m spacing case. The same pattern is observed for the 1,000 m spacing. Overall, the solution obtained even with a coarser grid is not that different than the solution found with the smallest grid spacing. The solution found by total enumeration using a coarser grid (e.g., 200 m) is close to solutions found using a finer grid. It would be possible to use the former solution as a starting point for the Nelder–Mead or simulated annealing approaches (Table 2).

The quality of the solution and the running time could be improved by combining several heuristics. For instance, besides using the center of the study region as the starting point of the simulated annealing heuristic, the use of the efficient variable neighborhood search technique (Hansen and Mladenovic 1997) is recommended. It would be interesting to combine the solution found using total enumeration with another search algorithm.

Comparison of the heuristics: Simultaneous addition
In the two-towers cases, Nelder–Mead uses 20 pairs of two potential cell tower locations as starting points, whereas 30 pairs of three potential cell towers are used in the three-towers cases. Using a greater number of starting points decreases the

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| Table 1 Results for the Case Study—Sequential Addition |
|---------------------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                             | TE              | SA finite P    | SA infinite P   | NM infinite P   |
| 1st tower                                  |                 |                |                 |                 |
| WCCP                                       | 0.87052         | 0.87052        | 0.87099         | 0.86920         |
| ΔCOV                                       | 1.6741          | 1.6741         | 1.729           | 1.519           |
| Time (min)                                 | 1,422.4         | 231.56         | 136.87          | 53.5            |
| Tower location (x)                         | 11,900          | 11,900         | 11,864.26       | 14,603.16       |
| Tower location (y)                         | 8,000           | 8,000          | 7,956.23        | 11,677.25       |
| 2nd tower                                  |                 |                |                 |                 |
| WCCP                                       | 0.88316         | 0.88316        | 0.878518        | 0.87834         |
| ΔCOV                                       | 3.1502          | 3.1502         | 2.608           | 2.587           |
| Time (min)                                 | 1,306.8         | 237.39         | 137.97          | 52.5            |
| Tower location (x)                         | 14,600          | 14,600         | 5,593.84        | 2,806.47        |
| Tower location (y)                         | 11,700          | 11,700         | 2,966.45        | 9,598.02        |
| 3rd tower                                  |                 |                |                 |                 |
| WCCP                                       | 0.89046         | 0.89046        | 0.89156         | 0.89098         |
| ΔCOV                                       | 4.003           | 4.003          | 4.1314          | 4.0643          |
| Time (min)                                 | 1,261.4         | 232.06         | 136.87          | 57.5            |
| Tower location (x)                         | 5,600           | 5,600          | 14,602.63       | 11,864.46       |
| Tower location (y)                         | 3,000           | 3,000          | 11,677.30       | 7,957.11        |

Improvements are calculated against the initial solution: 0.856189.
WCCP, weighted call completion probability.
likelihood that Nelder–Mead finds a local optimum. Overall, Nelder–Mead provides very poor results, not only because of its localized search but also because of its sensitivity to problems with a larger number of variables. Simulated annealing requires a greater number of iterations than the sequential case does, because two or three towers are being located simultaneously, requiring a greater effort from the algorithm. Simulated annealing returns slightly lower objective function values than the sequential case when using the finite set $P$ and slightly higher values when using the infinite set $P$. In the three-towers case, simulated annealing returns poor results when a solution is found from the finite set $P$. However, an optimal solution is found when using the infinite set $P$. Nevertheless, the running time for infinite (continuous) simulated annealing is very long (Table 3).

### Effect of topography
Topographic variation affects the solution, but its influence is lessened by the call completion probabilities for most sample points being equal to 1. If call completion probabilities are lower, topography would have a greater impact (equation [7]). With a higher tower height, as is expected, WCCP values are higher (WCCP values of 0.8788, 0.8841, and 0.8914 after the addition of the first, second and third tower, respectively). The limited improvement upon a 30-m-tower height is explained by the overall existing high call completion probabilities.

### Table 2  Sequential Addition: Effect of the Level of Discretization of $P$ Upon the Quality of the Solution, Using a Total Enumeration (TE) Algorithm

<table>
<thead>
<tr>
<th></th>
<th>50 m</th>
<th>100 m</th>
<th>150 m</th>
<th>200 m</th>
<th>500 m</th>
<th>1,000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st tower</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>WCCP</td>
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<td>0.87052</td>
<td>0.87087</td>
<td>0.87042</td>
<td>0.86975</td>
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<tr>
<td>ΔCOV</td>
<td>1.716</td>
<td>1.6741</td>
<td>1.715</td>
<td>1.663</td>
<td>1.58</td>
<td>1.58</td>
</tr>
<tr>
<td>Time (min)</td>
<td>4,838.1</td>
<td>1,422.4</td>
<td>593.17</td>
<td>387.01</td>
<td>59.57</td>
<td>15.35</td>
</tr>
<tr>
<td>Tower location (x)</td>
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<td>11,900</td>
<td>11,850</td>
<td>11,800</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>Tower location (y)</td>
<td>7,950</td>
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<td>7,950</td>
<td>8,000</td>
<td>8,000</td>
<td>8,000</td>
</tr>
<tr>
<td>2nd tower</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WCCP</td>
<td>0.88349</td>
<td>0.88316</td>
<td>0.88312</td>
<td>0.88202</td>
<td>0.87891</td>
<td>0.87815</td>
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<tr>
<td>ΔCOV</td>
<td>3.189</td>
<td>3.1502</td>
<td>3.1456</td>
<td>3.017</td>
<td>2.65</td>
<td>2.57</td>
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<tr>
<td>Time (min)</td>
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<td>1,306.8</td>
<td>762.5</td>
<td>307.62</td>
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<td>14,600</td>
<td>14,700</td>
<td>14,600</td>
<td>15,000</td>
<td>15,000</td>
</tr>
<tr>
<td>Tower location (y)</td>
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<td>11,700</td>
<td>11,600</td>
<td>11,500</td>
<td>12,000</td>
</tr>
<tr>
<td>3rd tower</td>
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<td></td>
<td></td>
<td></td>
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<td>WCCP</td>
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<td>0.89046</td>
<td>0.89013</td>
<td>0.88934</td>
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<td>ΔCOV</td>
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<td>3.87</td>
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<td>759.8</td>
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<td>5,500</td>
<td>5,600</td>
<td>5,500</td>
<td>6,000</td>
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<tr>
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<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
<td>3,000</td>
</tr>
</tbody>
</table>

WCCP, weighted call completion probability.
Conclusion

In this article, we address the additional cell tower location problem from the adaptive spatial sampling approach using kriging, capitalizing on available cell phone signal strength measurements, topography, and demand in the form of population counts. The numerical expression for the optimization of the WCCP function is explained, and a case study is presented. Because the objective function is highly nonlinear, we develop a simulated annealing and Nelder–Mead heuristic and obtain good quality solutions, in a much faster time frame than if total enumeration had been used. The results from the heuristics depend upon whether the set of candidate locations \( P \) is discretized or not, and on the level of discretization. Our case study illustrates the problems of locating one, two, and three additional cell towers in a rural section of Erie County, New York. Cell phone completion probabilities are derived from signal strength measurements and combined with population weights. Topography is reflected in the form of line-of-sight. Each time a tower is added, the call completion probabilities in the vicinity of the tower are updated accordingly. Results from the simultaneous addition of towers are not encouraging when using Nelder–Mead, but the quality of the solutions found using simulated annealing are better (but not than for the sequential case). Either the
optimal found in the sequential case is the overall optimal, or the simultaneous simulated annealing must be run for a longer time. Finally, the impact of tower height is limited; if the measured initial call completion probabilities were lower, we would experience a greater influence of that parameter.

The approach that we present here can easily be extended to problems of determining an optimal additional facility, contingent on the availability of some information about customer probability. The only changes would be the values of probabilities and the range of the covariogram. It would be realistic to add a new facility where the probability of visiting that facility would be a maximum. If customer probabilities were relatively low, there would be little incentive to add a new facility in that region. As is done within this article, customer probabilities could be weighted by population density.

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References


